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String Unification and Leptophobic Z' in Flipped SU(5)

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We summarize recent developments in the prediction for $\alpha_s(M_Z)$, self-consistent string unification and the dynamical determination of mass scales, and leptophobic Z' gauge bosons in the context of stringy flipped SU(5).

1. An oldie but goodie

Flipped SU(5) enthusiasts keep discovering hidden treasures, even after 10 years from its birth [1]. As is well known, the model attains its highest relevance in strings: efforts by several groups using different approaches have not (yet?) yielded appealing "string GUTs" [SO(10)]. Among level-one Kac-Moody models, only flipped SU(5) unifies SU(3) and SU(2), providing an explanation for the "LEP scale" [10^{16} GeV]. The discrepancy between "observed" and predicted unification scales – $M_{\rm LEP}\sim 10^{16}$ GeV versus $M_{\rm string}\sim 10^{18}$ GeV – seems to have only way out: extra intermediate-scale states [2]. This solution was realized early on in stringy flipped SU(5) [3]. Here we summarize how this scenario may be achieved in practice [4], including the prediction for $\alpha_s(M_Z)$ [5], and also discuss the latest "flipped" goodie: a leptophobic Z' [6].

2. Some basics first

Matter fields:
$F_{(10)}=\{Q,d^c,\nu^c\}$; $\bar f_{(\bar 5)}=\{L,u^c\}$; $l^c_{(1)}=e^c$ $(\times 3)$
$F_{(10)}=\{Q,d^c,\nu^c\}$; $F_{(\overline{10})}=\{\bar Q,\bar d^c,\bar\nu^c\}$
Higgs fields:
$H_{(10)}=\{Q_H,d^c_H,\nu^c_H\}$; $\bar H_{(\overline{10})}=\{Q_{\bar H},d^c_{\bar H},\nu^c_{\bar H}\}$
$h_{(5)}=\{H_2,H_3\}$; $\bar h_{(\bar 5)}=\{\bar H_2,\bar H_3\}$
GUT superpotential:

$$W_G=H\cdot H\cdot h+\bar H\cdot\bar H\cdot\bar h+F\cdot\bar H\cdot\phi+\mu h\bar h$$

The vevs $\langle\nu^c_H\rangle=\langle\nu^c_{\bar H}\rangle=M_U$ break $SU(5)\times U(1)$ down to $SU(3)\times SU(2)\times U(1)$.

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Doublet-triplet splitting:

$$h=\begin{pmatrix}H_2\\ H_3\end{pmatrix}\begin{array}{l}\text{electroweak symmetry breaking}\\ \text{mediates proton decay}\end{array}$$

$$\begin{array}{l}H\cdot H\cdot h\to d^c_H\left\langle\nu^c_H\right\rangle H_3\\ \bar H\cdot\bar H\cdot\bar h\to\bar d^c_H\left\langle\bar\nu^c_H\right\rangle\bar H_3\end{array}$$

The triplets get heavy, while the doublets remain light ("missing partner mechanism").
Yukawa superpotential:

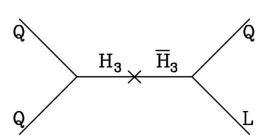
$$\lambda_d F\cdot F\cdot h+\lambda_u F\cdot\bar f\cdot\bar h+\lambda_e\bar f\cdot l^c\cdot h$$

Neutrino masses: The GUT couplings $F\cdot\bar f\cdot h\to m_u\nu\nu^c$, $F\cdot\bar H\cdot\phi\to\left\langle\nu^c_{\bar H}\right\rangle\nu^c\,\phi$ entail

$$M_\nu=\begin{array}{c c}& \begin{array}{ccc}\nu & \nu^c & \phi\end{array}\\ \begin{array}{c}\nu\\ \nu^c\\ \phi\end{array} & \begin{pmatrix}0 & m_u & 0\\ m_u & 0 & M_U\\ 0 & M_U & M\end{pmatrix}\end{array}$$

See-saw mechanism: $m_{\nu_{e,\mu,\tau}}\sim m^2_{u,c,t}/(M^2_U/M)$
Good for MSW mechanism, ν_τ hot dark matter, and (ν^c) baryogenesis.
Dimension-six proton decay: mediated by X,Y GUT gauge bosons, the mode $p\to e^+\pi^0$ may be observable at SuperKamiokande.
Dimension-five proton decay: very suppressed since no $H_3,\bar H_3$ mixing exists, even though $H_3,\bar H_3$ are heavy via doublet-triplet splitting,

$$\lambda_d F\cdot F\cdot h\supset QQH_3\qquad\lambda_u F\cdot\bar f\cdot\bar h\supset QL\bar H_3$$



3. Prediction for $\alpha_s(M_Z)$

Starting from the low-energy Standard Model gauge couplings, and evolving them from low to high energies, first α_2 and α_3 unify at M_{32} [5]

$$\frac{1}{\alpha_2} - \frac{1}{\alpha_5} = \frac{b_2}{2\pi} \ln \frac{M_{32}}{M_Z}$$
$$\frac{1}{\alpha_3} - \frac{1}{\alpha_5} = \frac{b_3}{2\pi} \ln \frac{M_{32}}{M_Z}$$

The hypercharge does not unify at M_{32}:

$$\frac{1}{\alpha_Y} - \frac{1}{\alpha_1'} = \frac{b_Y}{2\pi} \ln \frac{M_{32}}{M_Z}$$

Above M_{32} the gauge group is SU(5)×U(1). Stringy unification occurs at $M_{51} \geq M_{32}$ – there is an $M_{32}^{\max}$

$$\frac{1}{\alpha_Y} - \frac{1}{\alpha_5} = \frac{b_Y}{2\pi} \ln \frac{M_{32}^{\max}}{M_Z}$$

Solving for α_3, to lowest order:

$$\alpha_s(M_Z) = \frac{\frac{7}{3}\,\alpha}{5\sin^2\theta_W - 1 + \frac{11}{2\pi}\,\alpha \ln(M_{32}^{\max}/M_{32})}$$

compare with SU(5) where $M_{32} = M_{32}^{\max}$ [5]

$\alpha_s(M_Z)^{\text{Flipped SU(5)}} < \alpha_s(M_Z)^{\text{SU(5)}}$

What happens at next-to-leading order?

$\sin^2\theta_W \to \sin^2\theta_W - \delta_{\text{2loop}} - \delta_{\text{light}} - \delta_{\text{heavy}}$

Decreasing $\sin^2\theta_W$ increases $\alpha_s(M_Z)$ [avoid!]: $\delta_{\text{2loop}} \approx 0.0030$; $\delta_{\text{light}} \gtrsim 0$ (light SUSY thresholds); δ_{heavy} (GUT thresholds)

$$\delta_{\text{heavy}} = \frac{\alpha}{20\pi}\left[-6\ln\frac{M_{32}}{M_{H_3}} - 6\ln\frac{M_{32}}{M_{\bar{H}_3}} + 4\ln\frac{M_{32}}{M_V}\right]$$

Since there is no problem with proton decay, δ_{heavy} can be negative. We obtain $\alpha_s(M_Z)$ as low as 0.108 (see Fig. 1). However, decreasing M_{32} decreases the proton lifetime

$$\tau(p \to e^+\pi^0) \approx 1.5 \times 10^{33} \left(\frac{M_{32}}{10^{15}\,\text{GeV}}\right)^4 \left(\frac{0.042}{\alpha_5}\right)^2 \text{y}$$

The present lower bound $\tau(p \to e^+\pi^0)^{\exp} > 5.5 \times 10^{32}$ y implies $\alpha_s(M_Z) > 0.108$ (see Fig. 2). If $\alpha_s(M_Z) < 0.114$ then $p \to e^+\pi^0$ may be observable at SuperKamiokande (which should have a sensitivity of $\sim 10^{34}$ y). This is in contrast with minimal SU(5), where the preferred mode is $p \to \bar{\nu}K^+$.

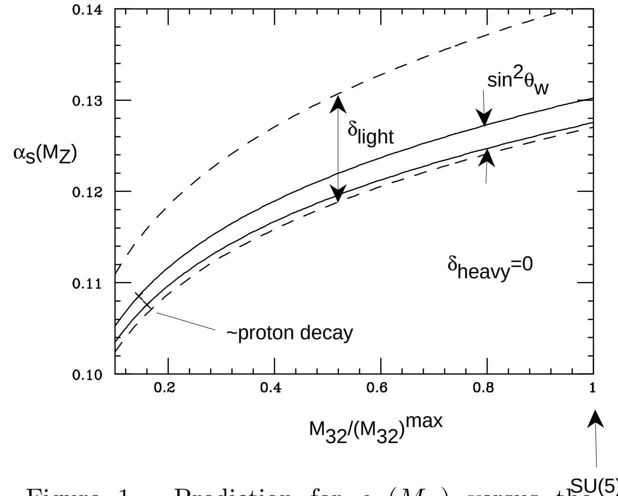


Figure 1. Prediction for $\alpha_s(M_Z)$ versus the SU(2)/SU(3) unification scale M_{32}.

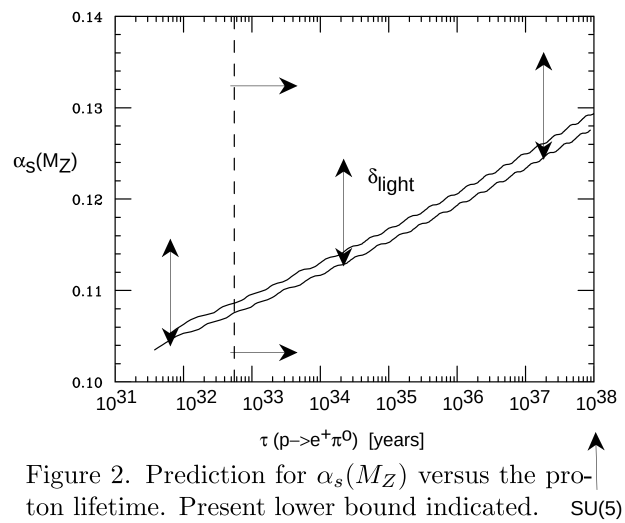


Figure 2. Prediction for $\alpha_s(M_Z)$ versus the proton lifetime. Present lower bound indicated.

4. Stringy Flipped SU(5)

String construction in fermionic formulation [3]
Gauge group: $G = G_{\text{observable}} \times G_{\text{hidden}} \times G_{\text{U(1)}}$
$G_{\text{observable}}$=SU(5)×U(1);
G_{hidden}=SO(10)×SU(4); $G_{\text{U(1)}}$=U(1)^5
Particle spectrum
Observable Sector:

$F^{\{0,1,2,3,4\}}[10]$ $\quad \bar{f}^{\{2,3,5\}}[\bar{5}]$ $\quad \ell^{c\,\{2,3,5\}}[1]$
$\bar{F}^{\{4,5\}}[\overline{10}]$
$h^{\{1,2,3,45\}}[5]$ $\quad \bar{h}^{\{1,2,3,45\}}[\bar{5}]$

Singlets: 20 charged under U(1)'s, 4 neutral

Hidden Sector:

$\mathrm{T}^{\{1,2,3\}}$	[10] of SO(10)
$\mathrm{D}^{\{1,2,3,4,5,6,7\}}$	[6] of SU(4)
$\widetilde{\mathrm{F}}^{\{1,2,3,4,5,6\}}$	[4] of SU(4)
$\overline{\widetilde{\mathrm{F}}}^{\{1,2,3,4,5,6\}}$	$[\bar{4}]$ of SU(4)

The $\widetilde{F}_i, \overline{\widetilde{F}}_j$ fields carry $\pm 1/2$ electric charges and exist only confined in hadron-like *cryptons*.

The cubic and non-renormalizable terms in the superpotential have been calculated [3], and more recently also the Kähler potential [7]. The properties of the Kähler potential illuminate the vacuum energy (which vanishes at tree level and possibly also at one loop) and determine the pattern of soft-supersymmetry-breaking masses, which has distinct experimental consequences [8].

5. String unification

Assume that

$$\mathrm{SU}(5) \times \mathrm{U}(1) \rightarrow \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$$

breaks as in Standard Flipped SU(5) case. Cancellation of $\mathrm{U_A}(1)$ is consistent with

$$M_{\mathrm{LEP}} \sim \langle \nu^c_H \rangle \sim 10^{15-16}\,\mathrm{GeV}$$

Correct $\sin^2\theta_W$ and α_3 obtained because of extra $\mathbf{10}$,$\overline{\mathbf{10}}$ present in string massless spectrum. String unification occurs at $M_{\mathrm{string}} \sim 10^{18}$ GeV. This requires $M_{10} \sim 10^{8-9}$ GeV, which can be generated via VEVs of hidden matter fields.

Dynamical Determination of Scales [4]

$\mathbf{4}, \bar{\mathbf{4}}$ affect running of U(1) down to $\Lambda_4 = M_{\mathrm{string}}\; e^{8\pi^2/g^2\beta_4}$, where $\beta_4 = -12 + \frac{1}{2}N_4 + N_6$; Λ_4 depends on string spectrum of $\mathbf{4}, \bar{\mathbf{4}}, \mathbf{6}$, and on actual decoupling of particles between M_{string} and Λ_4 [tricky]. Extra $\mathbf{10}, \overline{\mathbf{10}}$ affect running of SU(5)×U(1) down to M_{10}. Naively, if $M_{4,\bar{4}} \sim \Lambda_4$, a non-renormalizable term $(10)(\overline{10})(4)(\bar{4})\frac{1}{M}$ implies $M_{10} \sim \frac{\langle 4\bar{4}\rangle}{M} \sim \frac{\Lambda_4^2}{M}$. But in strings $M_{4,\bar{4}} \sim \Lambda_4 \ll M$ is very unlikely; $M_{4,\bar{4}} = 0$ is more natural. In the actual string model we have a quintic term [and $M_{4,\bar{4}} = 0$]: $(10)(\overline{10})(4)(\bar{4})\phi\frac{1}{M}$, where the cancellation of $\mathrm{U_A}(1)$ implies $\langle\phi\rangle/M \sim 1/10$. With massless flavors ($M_{4,\bar{4}} = 0$) one expects $\langle 4\bar{4}\rangle \sim \infty$. Aharony, et. al. studied $\mathrm{SU(N_c)}$ with N_f "massless" flavors with supersymmetry-breaking scalar masses [9]. Supersymmetry-breaking masses ($\tilde{m}$) yield finite condensates

$$\langle H\bar{H}\rangle \sim \left[\frac{N_c}{N_c - N_f}\;\frac{1}{\tilde{m}}\right]^{(N_c-N_f)/2(2N_c-N_f)}$$

In our case ($N_c = 4$, $N_f = 2$) we obtain

$$\langle 4\bar{4}\rangle \sim \Lambda_4^2\left(\frac{\tilde{m}}{\Lambda_4}\right)^{-1/3} \gg \Lambda_4^2\;,$$

and we can calculate M_{10} from first principles

$$M_{10} \sim \left(\frac{\Lambda_4}{M}\right)^2 \left(\frac{\tilde{m}}{\Lambda_4}\right)^{-1/3} \frac{\langle\phi\rangle}{M}\; M \sim 10^{8\to10}\,\mathrm{GeV}$$

This result allows self-consistent string unification. The results for the various scales as a function of $\alpha_s(M_Z)$ are shown in Fig. 3. The full evolution of the gauge couplings from the weak scale to the string scale is shown in Fig. 4 for the preferred choices of $\alpha_s(M_Z) = 0.116$ and $N_4 = 2$.

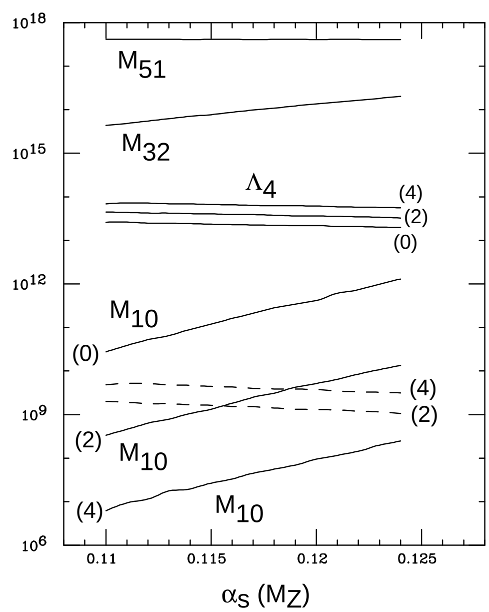


Figure 3. The calculated values of $M_{51} = M_{\mathrm{string}}$, $M_{32} = M_{\mathrm{LEP}}$, Λ_4, and M_{10}, as a function of $\alpha_s(M_Z)$ for $N_4 = 0, 2, 4$ (indicated in parenthesis). Dashed lines display estimates of the dynamical prediction for M_{10}.

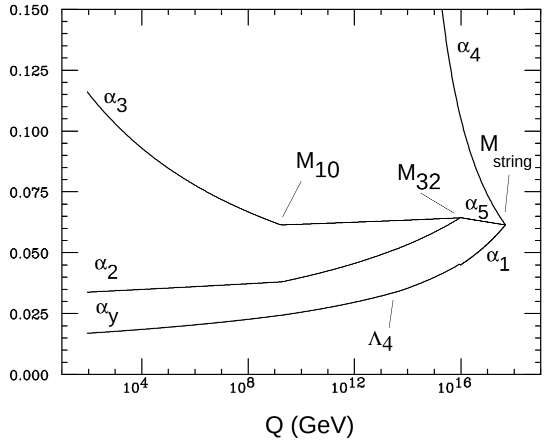


Figure 4. The running of the gauge couplings for $\alpha_s(M_Z) = 0.116$ and $N_4 = 2$. One obtains $M_{10} = 1.8 \times 10^9$ GeV, $M_{32} = 8.7 \times 10^{15}$ GeV, $M_{51} = 4.4 \times 10^{17}$ GeV, $\Lambda_4 = 3.9 \times 10^{13}$ GeV, and $g = 0.88$. This value of M_{10} agrees rather well with the dynamical prediction $M_{10} \sim 10^9$ GeV.

6. Leptophobic Z'

Motivation: Original "smoking gun" of string; R_b, R_c 'crisis' has revived interest in Z' models, although this time the Z' must not couple to leptons. Leptophobia is *natural* in flipped SU(5) [6]

$$10 = \{Q, d^c, \nu^c\}; \quad \bar{5} = \{L, u^c\}; \quad 1 = e^c$$

If the leptons are uncharged, most quarks may be charged under U$'$. Compare with regular SU(5) $10 = \{Q, u^c, e^c\}$; $\bar{5} = \{L, d^c\}$, where uncharged leptons imply uncharged quarks. Dynamic leptophobia (via RGE U(1) mixing) is also possible, as in the η-model in E_6 [10].

Any Z-Z' mixing shifts the usual Z couplings ($C^0_{V,A}$): $C_V = C^0_V + \theta(g_{Z'}/g_Z)\,C'_V$, $C_A = C^0_A + \theta(g_{Z'}/g_Z)\,C'_A$, where θ is the Z-Z' mixing angle (small); $g_Z, g_{Z'}$ are the Z, Z' gauge couplings; and $C'_{V,A}$ the fermion couplings to the Z'. In flipped SU(5) we have $[C'_V = Q_L + Q_R,\ C'_A = -Q_L + Q_R]$

	C^0_V	C^0_A	Q_L	Q_R	C'_V	C'_A
u	$\frac{1}{2} - \frac{4}{3}x_w$	$\frac{1}{2}$	c	0	c	$-c$
d	$-\frac{1}{2} + \frac{2}{3}x_w$	$-\frac{1}{2}$	c	c	$2c$	0

We can determine the first-order shifts in $\Gamma_{c\bar{c}}$, $\Gamma_{b\bar{b}}$, and $\Gamma_{\rm had}$, allowing for non-universal $c_{1,2,3}$ charges picked from

F_0	$-\frac{1}{2}$	$\bar{F}_4$	$\frac{1}{2}$	$\bar{f}_{2,3,5}$	0
F_1	$-\frac{1}{2}$	$\bar{F}_5$	0	$\ell^c_{2,3,5}$	0
F_2	0				
F_3	1				
F_4	$-\frac{1}{2}$				

This U$'$ charge space satisfies specific requirements: The leptons (in $\bar{f}_{2,3,5}, \ell^c_{2,3,5}$) are uncharged; one uncharged $(\mathbf{10}, \overline{\mathbf{10}})$ pair $(F_2, \bar{F}_5)$ so that U$'$ remains unbroken upon SU(5)$\times$U(1) breaking; Tr U$' = 0$ enforced; extra $(\mathbf{10}, \overline{\mathbf{10}})$ to allow string unification. The actual string model underlies these choices.

There are 13 possible charge assignments that can be made. Phenomenology demands $\Delta\Gamma_{\rm had} \lesssim 3\,$MeV, as the SM prediction and LEP agree well. Since $R_b^{\rm SM} = 0.2157$ and $R_b^{\rm exp} = 0.2202 \pm 0.0016$ (R_c fixed to SM value), we demand $\Delta R_b = 0.0030 - 0.0060$. Fig. 5 shows ΔR_b versus $\Delta\Gamma_{\rm had}$. An analogous plot for ΔR_c versus ΔR_b, demanding ΔR_c, ΔR_b shifts in opposite directions can be found in Ref. [6]. We should keep in mind that experimentally there appears to be a trend of R_c converging to the Standard Model prediction and R_b approaching it significantly.

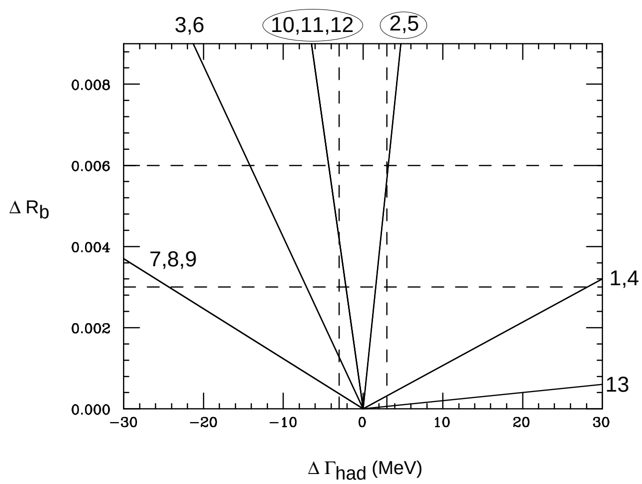


Figure 5. Correlated shifts in R_b and $\Gamma_{\rm had}$ for the various U$'$ charge assignment combinations. Dashed lines delimit the experimental limits on $\Delta\Gamma_{\rm had}$ and ΔR_b. Circled charge assignments (2,5,10,11,12) agree with experiment.

6.1. String scenario

Consider $G_{\mathrm{U}(1)} = \mathrm{U}_1 \times \mathrm{U}_2 \times \mathrm{U}_3 \times \mathrm{U}_4 \times \mathrm{U}_5$, with $\mathrm{Tr}\,\mathrm{U}_4 = 0$, $\mathrm{Tr}\,\mathrm{U}_{1,2,3,5} \neq 0$. The anomalous combination is $\mathrm{U}_\mathrm{A} = \mathrm{U}_1 - 3\mathrm{U}_2 + \mathrm{U}_3 + 2\mathrm{U}_5$, with three orthogonal traceless combinations: $\mathrm{U}'_1 = \mathrm{U}_3 + 2\mathrm{U}_5$; $\mathrm{U}'_2 = \mathrm{U}_1 - 3\mathrm{U}_2$; $\mathrm{U}'_3 = 3\mathrm{U}_1 + \mathrm{U}_2 + 4\mathrm{U}_3 - 2\mathrm{U}_5$. The lepton charges under $\mathrm{U}'_1, \mathrm{U}'_2, \mathrm{U}'_3$ are $\bar{f}_{2,5}, \ell^c_{2,5}$: $(0, \frac{3}{2}, -\frac{1}{2})$; $\bar{f}_3, \ell^c_3 : (\frac{3}{2}, 0, 1)$.
There is a *unique* U' that is leptophobic

$$\mathrm{U}' \propto 2\mathrm{U}'_1 - \mathrm{U}'_2 - 3\mathrm{U}'_3 \propto \mathrm{U}_1 + \mathrm{U}_3 - \mathrm{U}_5$$

and by construction $\mathrm{Tr}\,\mathrm{U}' = 0$. Higgs fields charged under U' exist (Z-Z' mixing). The D- and F-flatness conditions may be satisfied, leaving U' unbroken, but breaking the hidden group. Model building: F_4 should contain 3rd generation (top Yukawa); $F_2, \bar{F}_5$ neutral under U': symmetry breaking; $\bar{F}_4$: string unification; R_b, R_c inputs: four charge assignments allowed

	c_1	c_2	c_3
(2)	0	$-\frac{1}{2}$	1
(5)	$-\frac{1}{2}$	0	1
(11)	$-\frac{1}{2}$	1	$-\frac{1}{2}$
(12)	$-\frac{1}{2}$	$-\frac{1}{2}$	1

Unlike any considered before. Unnatural? Obtained from string! Top-quark Yukawa coupling, and R_b, R_c select scenario (11) uniquely

$$\Delta R_b \approx 0.0042 \left(\frac{\Delta\Gamma_{\mathrm{had}}}{-3\,\mathrm{MeV}}\right), \quad \Delta R_c \approx -0.76\,\Delta R_b \ .$$

Dynamics: Running of U' from M_Z up looks good: $b' = \frac{16}{3}$ (c.f. $b_Y = \frac{33}{5}$). Sufficiently small Z-Z' mixing appears to require radiative U' symmetry breaking via singlet $\langle\phi\rangle$.

6.2. Experimental prospects

Z' width and branching ratios for preferred case:

(11)	C'_V	C'_A	$B(Z' \to q\bar{q})$
u	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{18}$
d	-1	0	$\frac{1}{9}$
c	1	1	$\frac{2}{9}$
s	2	0	$\frac{4}{9}$
t	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{18}$
b	-1	0	$\frac{1}{9}$

$$\frac{\Gamma_{Z'}}{M_{Z'}} \approx 0.033 \left(\frac{g_{Z'}}{g_Z}\right)^2 \ \text{[narrow]}$$

Experimental limits:

$$\frac{\widehat{\sigma}(u\bar{u} \to Z')}{\widehat{\sigma}(u\bar{u} \to Z')_{\mathrm{SM}}} \approx 0.58 \left(\frac{g_{Z'}}{g_Z}\right)^2$$

$$\frac{\widehat{\sigma}(d\bar{d} \to Z')}{\widehat{\sigma}(d\bar{d} \to Z')_{\mathrm{SM}}} \approx 0.90 \left(\frac{g_{Z'}}{g_Z}\right)^2$$

Average up/down; multiply by $\frac{B(Z' \to jj)}{B(Z' \to jj)_{\mathrm{SM}}} \approx 1.4$,

$$\sigma(p\bar{p} \to Z' \to jj) \approx \left(\frac{g_{Z'}}{g_Z}\right)^2 \sigma(p\bar{p} \to Z' \to jj)_{\mathrm{SM}}$$

Only limit from UA2: $M_{Z'} > 260\,\mathrm{GeV}$, but only if $g_{Z'} = g_Z$.

Z' contributes to top-quark cross section (see Fig. 3 in Ref. [6]) at a level that may be observable if $M_{Z'} \sim 500\,\mathrm{GeV}$. Parity-violating spin asymmetries at RHIC may also show deviations from Standard Model expectations because of the t-channel exchange of our parity-violating Z'.

In sum, flipped SU(5) continues to provide unsolicited solutions to unanticipated problems, as evidenced most recently by the self-consistent string unification and the possible existence of a leptophobic Z' gauge boson.

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